IIT – JAM (PHYSICS) - 2022

Question paper and Solutions

Section A: Q.1 – Q.10 Carry ONE mark each.

1. The equation $z^2+(\bar{z})^2=4$ in the complex plane (where $\bar{z}$ is the complex conjugate of z) represents

   a) Ellipse B) Hyperbola c) Circle of radius 2 d) Circle of radius 4

2. A rocket (S') moves at a speed $\frac{c}{2}m/s$ along the positive x-axis, where c is the speed of light. When it crosses the origin, the clocks attached to the rocket and the one with a stationary observer (S) located at x = 0 are both set to zero. If S observes an event at (x,t), the same event occurs in the S' frame at

   $a)\ x' = \frac{2}{\sqrt{3}}(x-\frac{ct}{2})\, and\, t' = \frac{2}{\sqrt{3}}(t-\frac{x}{2c})$ $b)\ x' = \frac{2}{\sqrt{3}}(x+\frac{ct}{2})\, and\, t' = \frac{2}{\sqrt{3}}(t-\frac{x}{2c})$

   $c)\ x' = \frac{2}{\sqrt{3}}(x-\frac{ct}{2})\, and\, t' = \frac{2}{\sqrt{3}}(t+\frac{x}{2c})$ $d)\ x' = \frac{2}{\sqrt{3}}(x+\frac{ct}{2})\, and\, t' = \frac{2}{\sqrt{3}}(t+\frac{x}{2c})$

3. Consider a classical ideal gas of N molecules in equilibrium at temperature T. Each molecule has two energy levels, $-\varepsilon\, and\, \varepsilon$. The mean energy of the gas is

   a) 0 $b)\ N\varepsilon \tanh(\frac{\varepsilon}{k_B T})$ c) $-N\varepsilon \tanh(\frac{\varepsilon}{k_B T})$ d) $\frac{\varepsilon}{2}$

4. At a temperature T, let β and k denote the volume expansivity and isothermal compressibility of a gas, respectively. Then $\frac{\beta}{k}$ is equal to

   $a)\left(\frac{\partial P}{\partial T}\right)_v$ $b)\left(\frac{\partial P}{\partial V}\right)_T$ $c)\left(\frac{\partial T}{\partial P}\right)_V$ $d)\left(\frac{\partial T}{\partial V}\right)_P$

5. The resultant of the binary subtraction 1110101 − 0011110 is

   a) 1001111 b) 1010111 c) 1010011 d) 1010001

6. Consider a particle trapped in a three-dimensional potential well such that $U(x,y,z)=0\ for\ 0\le x\le a,\ 0\le y\le a,\ 0\le z\le a, and\ U(x,y,z)=\infty$ everywhere else. The degeneracy of the 5th excited state is

   a) 1 b) 3 c) 6 d) 9

7. A particle of mass m and angular momentum L moves in space where its potential energy is U(r)= k r^2 (k>0) and r is the radial coordinate. If the particle moves in a circular orbit, then the radius of the orbit is

$a)\left[\dfrac{L^2}{mk}\right]^{\frac{1}{4}}$ $b)\left[\dfrac{L^2}{2mk}\right]^{\frac{1}{4}}$ $c)\left[\dfrac{2L^2}{mk}\right]^{\frac{1}{4}}$ $d)\left[\dfrac{4L^2}{mk}\right]^{\frac{1}{4}}$

8. Consider a two-dimensional force field $\vec{F}(x,y) = (5x^2 + ay^2 + bxy)\,x + (4x^2 + 4xy + y^2)\,y$. If the force field is conservative, then the values of a and b are

a) a = 2 and b = 4 b) a = 2 and b = 8

c) a = 4 and b = 2 d) a = 8 and b = 2

9. Consider an electrostatic field $\vec{E}$ in a region of space. Identify the **INCORRECT** statement

a) The work done in moving a charge in a closed path inside the region is zero

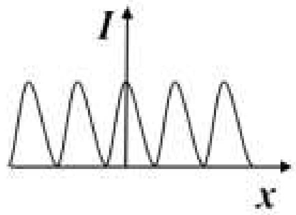
b) The curl of $\vec{E}$ is zero

c) The field can be expressed as the gradient of a scalar potential

d) The potential difference between any two points in the region is always zero

10. Which one of the following figures correctly depicts the intensity distribution for Fraunhofer diffraction due to a single slit? Here, ***x*** denotes the distance from the centre of the central fringe and ***I*** denotes the intensity.

(***a***)



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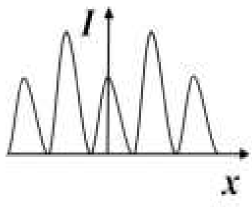

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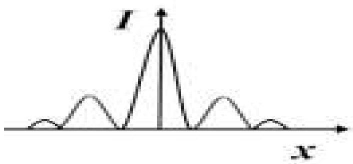

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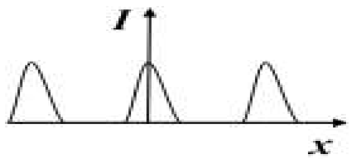


SECTION – C

Q.11 – Q.30 Carry TWO marks each.

11. The function $f(x) = e^{\sin x}$ is expanded as a Taylor series in x, around x = 0, in the form $f(x) = \sum_{n=0}^{\infty} a_n x^n$. The value of $a_0 + a_1 + a_2$ *is*

a) 0 b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) 5

12. Consider a unit circle C in the xy plane, centered at the origin. The value of the integral $\oint [(\sin x - y)\,dx - (\sin y - x)\,dy]$ over the circle C, traversed anticlockwise, is

a) 0 b) 2π c) 3π d) 4π

13. The current through a series RL circuit, subjected to a constant emf ε, obeys $L\frac{di}{dt} + iR = \varepsilon$. Let $L = 1mH$, $R = 1\,k\Omega$ and $\varepsilon = 1\,V$. The initial condition is $i(0) = 0$. At $t = 1\,\mu s$, the current in mA is

$a)\,1-2e^{-2}$ $b)\,1-2e^{-1}$ $c)\,1-e^{-1}$ $d)\,2-2e^{-1}$

14. An ideal gas in equilibrium at temperature T expands isothermally to twice its initial volume. If ΔS, ΔU and ΔF denote the changes in its entropy, internal energy and Helmholtz free energy respectively, then

a) $\Delta S < 0, \Delta U > 0, \Delta F < 0$ b) $\Delta S > 0, \Delta U = 0, \Delta F < 0$

c) $\Delta S < 0, \Delta U = 0, \Delta F > 0$ d) $\Delta S > 0, \Delta U > 0, \Delta F = 0$

15. In a dilute gas, the number of molecules with free path length $\geq x$ is given by $N(x) = N_0 e^{-x/\lambda}$, where N_o is the total number of molecules and λ is the mean free path. The fraction of molecules with free path lengths between λ and 2λ

a) $\frac{1}{e}$ b) $\frac{e}{e-1}$ c) $\frac{e^2}{e-1}$ d) $\frac{e-1}{e^2}$

16. Consider a quantum particle trapped in a one-dimensional potential well in the region $[-L/2 < x < L/2]$, with infinitely high barriers at $x = -L/2 \, and \, x = L/2$. The stationary wave function for the ground state is $\psi(x) = \sqrt{\frac{2}{L}}\cos(\frac{\pi x}{L})$ The uncertainties in momentum and position satisfy

$a)\ \Delta p = \frac{\pi\hbar}{L}\ and\ \Delta x = 0$ $\qquad$ $b)\ \Delta p = \frac{2\pi\hbar}{L}\ and\ 0 < \Delta x < \frac{L}{2\sqrt{3}}$

$c)\ \Delta p = \frac{\pi h}{L}\ and\ \Delta x > \frac{L}{2\sqrt{3}}$ $\qquad$ $d)\ \Delta p = 0\ and\ \Delta x = \frac{L}{2}$

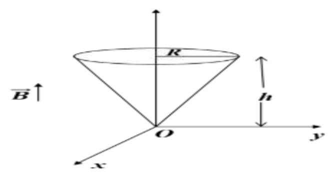
17. Consider a particle of mass m moving in a plane with a constant radial speed $\dot{r}$ and a constant speed $\dot{\theta}$. The acceleration of the particle in (r,θ) coordinates

$a)\ 2r\dot{\theta}^2\hat{r} - \dot{r}\dot{\theta}\theta$ $\qquad$ $b)\ -r\dot{\theta}^2\hat{r} + 2\dot{r}\dot{\theta}\hat{\theta}$ $\qquad$ $c)\ \ddot{r}\hat{r} + r\ddot{\theta}\theta$ $\qquad$ $d)\ \ddot{r}\theta\hat{r} + r\ddot{\theta}\theta$

18. A planet of mass m moves in an elliptical orbit. Its maximum and minimum distances from the Sun are R and r, respectively. Let G denote the universal gravitational constant, and M the mass of the Sun Assuming M >>m, the angular momentum of the planet with respect to the center of the Sun is

$a)\ m\sqrt{\frac{2GMRr}{(R+r)}}$ $\qquad$ $b)\ m\sqrt{\frac{GMRr}{2(R+r)}}$ $\qquad$ $c)\ m\sqrt{\frac{GMRr}{(R+r)}}$ $\qquad$ $d)\ 2m\sqrt{\frac{2GMRr}{(R+r)}}$

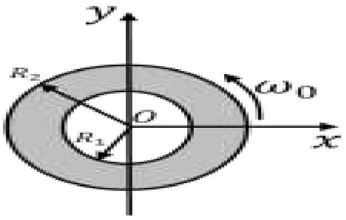
19. Consider a conical region of height h and base radius R with its vertex at the origin. Let the outward normal to its base be along the positive z-axis, as shown in the figure. A uniform magnetic field, $\vec{B} = B_0\hat{z}$ exists everywhere. Then the magnetic flux through the base (Φ_b) and that through the curved surface of the cone (Φ_c) are



$a)\ \phi_b = B_0\pi R^2;\ \phi_c = 0$ $\qquad$ $b)\ \phi_b = -\frac{1}{2}B_0\pi R^2;\ \phi_c = \frac{1}{2}B_0\pi R^2$

$c)\ \phi_b = 0;\ \phi_c = -B_0\pi R^2$ $\qquad$ $d)\ \phi_b = B_0\pi R^2;\ \phi_c = -B_0\pi R^2$

20. Consider a thin annular sheet, lying on the *xy*-plane, with R_1 *and* R_2 as its inner and outer radii, respectively. If the sheet carries a uniform surface-charge density σ and spins about the origin 0 with a constant angular velocity $\vec{\omega} = \omega_0 \hat{z}$ then, the total current flow on the sheet is

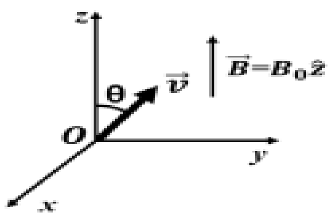


$a)\ \dfrac{2\pi\sigma\omega_0(R_2^3 - R_1^3)}{3}$ $\quad b)\ \sigma\omega_0(R_2^3 - R_1^3)$ $\quad c)\ \dfrac{\pi\sigma\omega_0(R_2^3 - R_1^3)}{3}$ $\quad d)\ \dfrac{2\pi\sigma\omega_0(R_2 - R_1)^3}{3}$

21. A radioactive nucleus has decay constant λ and its radioactive daughter nucleus has a decay constant 10λ. At time t = 0, N_o is the number of parent nuclei and there are no daughter nuclei present. $N_1(t)$ and $N_2(t)$ are the number of parent and daughter nuclei present at time t, respectively. The ratio $N_2(t)/N_1(t)$ is

$a)\dfrac{1}{9}\left[1-e^{-9\lambda t}\right]$ $\quad b)\dfrac{1}{10}\left[1-e^{-10\lambda t}\right]$ $\quad c)\left[1-e^{-10\lambda t}\right]$ $\quad d)\left[1-e^{-9\lambda t}\right]$

22. A uniform magnetic field $\vec{B} = B_0 z$, where $B_0 > 0$ exists as shown in the figure. A charged particle of mass *m* and charge *q* (*q* > *0*) is released at the origin, in the *yz*-plane, with a velocity $\vec{v}$ directed at an angle $\theta = 45^\circ$ with respect to the positive z-axis. Ignoring gravity, which one of the following is **TRUE**.



a) The initial acceleration $\vec{a} = \dfrac{qvB_0}{\sqrt{2}\ m} x$

b) The initial acceleration $\vec{a} = \dfrac{qvB_0}{\sqrt{2}\ m} y$

c) The particle moves in a circular path

d) The particle continues in a straight line with constant speed

23. For an ideal intrinsic semiconductor, the Fermi energy at 0 K

a) lies at the top of the valence band

b) lies at the bottom of the conduction band

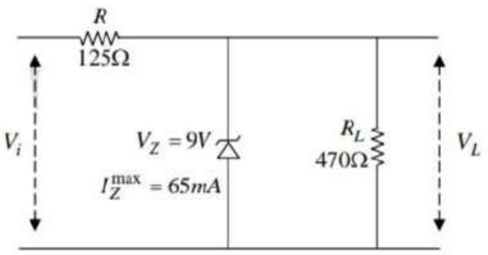
c) lies at the center of the band gap

d) lies midway between center of the band gap and bottom of the conduction band

24. A circular loop of wire with radius R is centered at the origin of the xy-plane. The magnetic field at a point within the loop is, $\vec{B}(\rho,\phi,z,t)=k\,\rho^3 t^3\,\hat{z}$, where k is a positive constant of appropriate dimensions. Neglecting the effects of any current induced in the loop, the magnitude of the induced *emf* in the loop at time t is

a) $\frac{6\pi k t^2 R^5}{5}$ b) $\frac{5\pi k t^2 R^5}{6}$ c) $\frac{3\pi k t^2 R^5}{2}$ d) $\frac{\pi k t^2 R^5}{2}$

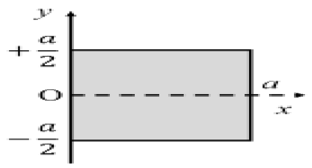
25. For the given circuit, $R=125\ \Omega, R_L=470\,\Omega, V_z=9V, and\ I_z^{\max}=65mA$ The minimum and maximum values of the input voltage $(V_i^{\min}\ and\ V_i^{\max})$ for which the Zener diode will be in the 'ON' state are



a) $V_i^{\min}=9.0V\ and\ V_i^{\max}=11.4V$ b) $V_i^{\min}=9.0V\ and\ V_i^{\max}=19.5V$

c) $V_i^{\min}=11.4V\ and\ V_i^{\max}=15.5V$ d) $V_i^{\min}=11.4V\ and\ V_i^{\max}=19.5V$

26. A square laminar sheet with side a and mass M, has mass per unit area given by $\sigma(x)=\sigma_0\left[1-\frac{x}{a}\right]$, (see figure). Moment of inertia of the sheet about y-axis is



a) $\frac{Ma^2}{2}$ b) $\frac{Ma^2}{4}$ c) $\frac{Ma^2}{6}$ d) $\frac{Ma^2}{12}$

27. A particle is subjected to two simple harmonic motions along the x and y axes, described by $x(t)=a\sin(2\omega t+\pi)\ and\ y(t)=2a\sin(\omega t)$. The resultant motion is given by

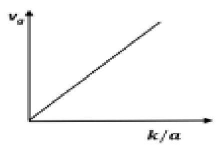
a) $\frac{x^2}{a^2}+\frac{y^2}{4a^2}=1$ b) $x^2+y^2=1$ c) $y^2=x^2(1-\frac{x^2}{4a^2})$ d) $x^2=y^2(1-\frac{y^2}{4a^2})$

28. For a certain thermodynamic system, the internal energy U=PV and P is proportional to T^2. The entropy of the system is proportional to

$a) UV$ $\qquad$ $b) \sqrt{\frac{U}{V}}$ $\qquad$ $c) \sqrt{\frac{V}{U}}$ $\qquad$ $d) \sqrt{UV}$

29. The dispersion relation for certain type of waves is given by $\omega = \sqrt{k^2 + a^2}$, where k is the wave vector and a is a constant. Which one of the following sketches represents the v_g, group velocity?

(***a***)



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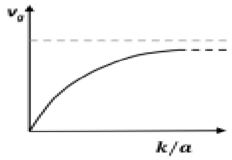

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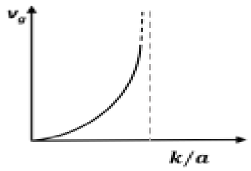

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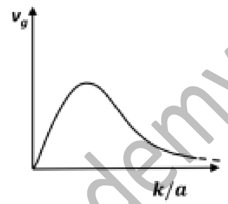


30. Consider a binary number with m digits, where m is an even number. This binary number has alternating 1's and 0's, with digit 1 in the highest place value. The decimal equivalent of this binary number

$a) 2^m - 1$ $\qquad$ $b) \frac{(2^m - 1)}{3}$ $\qquad$ c) $\frac{(2^{m+1} - 1)}{3}$ $\qquad$ $d) \frac{2}{3}(2^m - 1)$

31. Consider the 2 × 2 matrix $M = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix}$ where a,b >0. Then.

a) M is a real symmetric matrix $\qquad$ b) One of the eigenvalues of M is greater than b

c) One of the eigenvalues of M is negative $\qquad$ d) Product of eigenvalues of M is b

32. In the Compton scattering of electrons, by photons incident with wavelength λ,

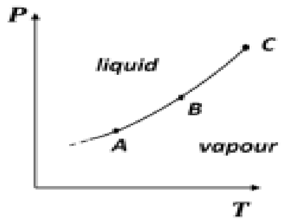
a) $\frac{\Delta\lambda}{\lambda}$ is independent of λ

b) $\frac{\Delta\lambda}{\lambda}$ increases with decreasing

c) there is no change in photon's wavelength for all angles of deflection of the photon

d) $\frac{\Delta\lambda}{\lambda}$ increases with increasing angle of deflection of the photon

33. The figure shows a section of the phase boundary separating the vapour (1) and liquid (2) states of water in the $P-T$ plane. Here, C is the critical point. μ_1, v_1 and s_1 are the chemical potential, specific volume and specific entropy of the vapour phase respectively, while μ_2, v_2 and s_2 respectively denote the same for the liquid phase. Then



a) $\mu_1 = \mu_2$ along AB

b) $v_1 = v_2$ along AB

c) $s_1 = s_2$ along AB

d) $v_1 = v_2$ at the point C

34. A particle is executing simple harmonic motion with time period T. Let x,v and a denote the displacement, velocity and acceleration of the particle, respectively, at time *t*. Then,

a) $\frac{aT}{x}$ does not change with time

b) $(aT + 2\pi v)$ does not change with time

c) x and v are related by an equation of a straight line

d) v and a are related by an equation of an ellipse

35. A linearly polarized light beam travels from origin to point A $(1,0,0)$. At the point A, the light is reflected by a mirror towards point B $(1, -1, 0)$. A second mirror located at point B then reflects the light towards point C $(1, -1, 1)$. Let $n(x, y, z)$ represent the direction of polarization of light at (x, y, z).

a) If $n(0, 0, 0) = y$, then $n(1, -1, 1) = x$

b) If $n(0, 0, 0) = \hat{z}$, then $n(1, -1, 1) = y$

c) If $n(0, 0, 0) = y$, then $n(1, -1, 1) = y$

d) If $n(0, 0, 0) = \hat{z}$, then $n(1, -1, 1) = x$

36. Let (r, θ) denote the polar coordinates of a particle moving in a plane. If $\hat{r}$ *and* θ represent the corresponding unit vectors, then

a) $\frac{d\hat{r}}{d\theta} = \theta$

b) $\frac{d\hat{r}}{dr} = -\theta$

c) $\frac{d\theta}{d\theta} = -\hat{r}$

d) $\frac{d\theta}{dr} = \hat{r}$

37. The electric field associated with an electromagnetic radiation is given by

$E = a(1+\cos w_1 t)\cos w_2 t$ Which of the following frequencies are present in the field?

$a)\ \omega_1$ $b)\ \omega_1+\omega_2$ $c)\ |\omega_1-\omega_2|$ $d)\ \omega_2$

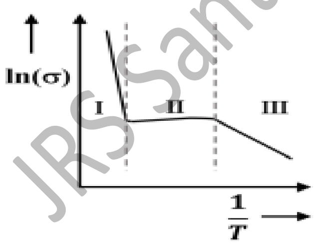
38. A string of length L is stretched between two points $x = 0\, and\, x = L$ and the endpoints are rigidly clamped. Which of the following can represent the displacement of the string from the equilibrium position?

$a)\, x\cos\left(\frac{\pi x}{L}\right)$ $b)\ x\sin\left(\frac{\pi x}{L}\right)$ c) $x\left(\frac{x}{L}-1\right)$ $d)\, x\left(\frac{x}{L}-1\right)^2$

39. The Boolean expression $Y = \overline{PQ}R + Q\overline{R} + \overline{P}QR + PQR$ simplifies to

$a)\,\overline{P}R+Q$ b) $PR+\overline{Q}$ $c)\,P+R$ $d)\,Q+R$

40. For an n-type silicon, an extrinsic semiconductor, the natural logarithm of normalized conductivity (σ) is plotted as a function of inverse temperature. Temperature interval-I corresponds to the intrinsic regime, interval-II corresponds to saturation regime and interval-III corresponds to the freeze-out regime, respectively. Then



a) the magnitude of the slope of the curve in the temperature interval-I is proportional to the bandgap, E_d

b) the magnitude of the slope of the curve in the temperature interval-III is proportional to the ionization energy of the donor, E_d

c) in the temperature interval-II, the carrier density in the conduction band is equal to the density of donors.

e) in the temperature interval-III, all the donor levels are ionized

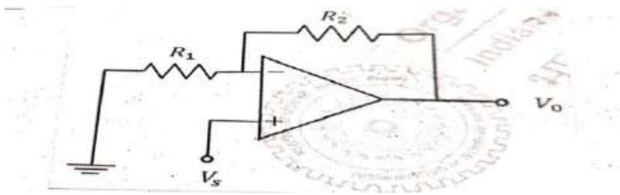
SECTION – C

NUMERICAL ANSWER TYPE (NAT)

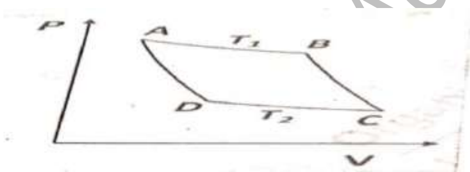
Section C: Q.41 – Q.50 Carry ONE mark each.

41. The integral $\iint (x^2+y^2)\,dxdy$ over the area of a disk of radius 2 in the xy plane is ___ π .

42. For the given operational amplifier circuit $R_1 = 120\Omega,\ R_2 = 1.5\ k\Omega$ and $V_s = 0.6V$, then the Output current I_0 is ____________ mA.



43. For an ideal gas, AB and CD are two isothermals at temperatures T_1 *and* $T_2 (T_1 > T_2)$, respectively. AD and BC represent two adiabatic paths as shown in figure. Let V_A, V_B, V_c *and* V_D be the volumes of the gas at A, B, C and D respectively. If $\frac{V_C}{V_B} = 2;$ *then* $\frac{V_D}{V_A} =$ ___________.



44. A satellite is revolving around the Earth in a closed orbit. The height of the satellite above Earth's surface at perigee and apogee are 2500 km and 4500 km, respectively. Consider the radius of the Earth to be 6500 km. The eccentricity of the satellite's orbit is ____ (Round off to 1 decimal place).

45. Three masses $m_1 = 1,\ m_2 = 2$ *and* $m_3 = 3$ are located on the x-axis such that their center of mass is at x = 1 . Another mass $m_4 = 4$ is placed at x_0 and the new center of mass is at x = 3. The value of x_0 is _____

46. A normal human eye can distinguish two objects separated by 0.35 m when viewed from a distance of 1.0 km. The angular resolution of eye is ________seconds (Round off to the nearest integer).

47. A rod with a proper length of $3m$ moves along x-axis, making an angle of 30° with respect to the x - axis. If its speed is $\frac{c}{2}m/s$, where c is the speed of light, the change in length due to Lorentz contraction is ____m (Round off to 2 decimal places). $[Use\, c = 3\times10^{8}\, m/s]$

48. Consider the Bohr model of hydrogen atom. The speed of an electron in the second orbit

$(n=2)\, is$ ______$\times10^{6}\, m/$ (Round off to 2 decimal places).

$[Use\, h = 6.63\times10^{-34}\, Js,\; e = 1.6\times10^{-19}\, C,\; \in_0 = 8.85\times10^{-12}\, C^2m^2/N]$

49. Consider a unit circle C in the xy plane with center at the origin. The line integral of the vector field, $\vec{F}(x,y,z) = -2yx - 3zy + xz,$ taken anticlockwise over C _____π.

50. Consider a p-n junction at T= 300 K. The saturation current density at reverse bias is $-20\mu A/cm^2$ For this device, a current density of magnitude $10\,\mu A/cm^2$. is realized with a forward bias voltage, V_F. The same magnitude of current density can also be realized with a reverse bias voltage, $V_{R.}$. The value of $|V_F/V_R|$ is _______ (Round off to 2 decimal places).

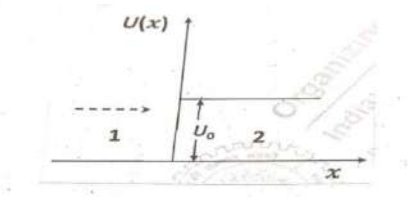
51. Consider the second order ordinary differential equation, $y''+4y'+5y=0.$ If $y(0)=0\, and\, y'(0)=1,$ then the value of $y\;(\pi/2)$ is __________ (Round off to 3 decimal places).

52. A box contains a mixture of two different ideal monatomic gases, 1 and 2, in equilibrium at temperature T. Both gases are present in equal proportions. The atomic mass for gas 1 is $m,$ while the same for gas 2 is $2\,m.$ If the *rms* speed of a gas molecule selected at random is

$v_{rms} = x\sqrt{\frac{k_BT}{m}}$, then x is __________ (Round off to 2 decimal places).

53. A hot body with constant heat capacity 800 J/K at temperature 925 K is dropped gently into a vessel containing 1kg of water at temperature 300 K and the combined system is allowed to reach equilibrium. The change in the total entropy ΔS is _________ J/K (Round off to 1 decimal place). Take the specific heat capacity of water to be 4200 $J/kg\,K$. Neglect any loss of heat to the vessel and air and change in the volume of water.]

54. Consider an electron with mass m and energy E moving along the x-axis towards a finite step potential of height U_o as shown in the figure. In region **1** $(x<0)$ the momentum of the electron is $p_1=\sqrt{2mE}$. The reflection coefficient at the barrier is given by $R=\left(\dfrac{p_1-p_2}{p_1+p_2}\right)^2$, where p_2 is the momentum in region **2** .If, in the limit $E>>U_0$ $R\approx\dfrac{U_0^{\,2}}{nE^2}$ then the integer n is ___.



55. A current density for a fluid flow is given by, $\vec{\mathrm{J}}(x,y,z,t)=\dfrac{8e^{t}}{(1+x^2+y^2+z^2)}x$. At time t=0, the mass density $\rho(x,y,z,0)=1$. Using the equation of continuity, $\rho(1,1,1,1)$ is found to be _______ (Round off to 2 decimal places).

56. The work done in moving a $-5\mu C$ charge in an electric field $\vec{E}=(8r\sin\theta\;\hat{r}+4r\cos\theta\;\theta)V/m$, from a point $A(r,\theta)=(10;\frac{\pi}{6})$ to a point $B(r,\theta)=(10;\frac{\pi}{2})$ is ______ mJ.

57. A pipe of 1 m length is closed at one end. The air column in the pipe resonates at its fundamental frequency of 400 Hz. The number of nodes in the sound wave formed in the pipe is _____.

[Speed of sound = 320 m/s]

58. The critical angle of a crystal is 30°. Its Brewster angle is ______ degrees (Round off to the nearest integer).

59. In an LCR series circuit, a non-inductive resistor of 150 Ω, a coil of 0.2 H inductance and negligible resistance, and a 30 μF capacitor are connected across an ac power source of 220 V, 50 Hz. The power loss across the resistor is ____W (Round off to 2 decimal places).

60. A charge q is uniformly distributed over the volume of a dielectric sphere of radius a. If the dielectric constant $\varepsilon_r=2$, then the ratio of the electrostatic energy stored inside the sphere to that stored outside is_______ (Round off to 1 decimal place).

Question No.	Key	Question No.	Key	Question No.	Key/Range (KY)
1	B	**24**	A	**47**	-0.31 to -0.29 or 0.29 to 0.31
2	A	**25**	D	**48**	1.08 to 1.10
3	C	**26**	C	**49**	2 to 2
4	A	**27**	D	**50**	0.57 to 0.61
5	B	**28**	D	**51**	0.041 to 0.045
6	C	**29**	B	**52**	1.50 to 1.50 or 1.57 to 1.59
7	B	**30**	D	**53**	537.5 to 537.7 or 549.8 to 550.2
8	B	**31**	A, B, C	**54**	16 to 16
9	D	**32**	B, D	**55**	2.70 to 2.74
10	C	**33**	A, D	**56**	-1 or 1
11	C	**34**	A, D	**57**	Marks to All
12	B	**35**	A, B	**58**	27 to 27 or 63 to 63
13	C	**36**	A, C	**59**	297 to 299
14	B	**37**	B, C, D	**60**	0.1 to 0.1
15	D	**38**	B, C, D		
16	Marks to All	**39**	D		
17	B	**40**	A, B, C		
18	A	**41**	8 to 8		
19	D	**42**	5 to 5		
20	Marks to All	**43**	2 to 2		
21	A	**44**	0.1 to 0.1		
22	A	**45**	6 to 6		
23	C	**46**	71 to 73		

IIT JAM 2022 PHYSICS

Answers with Explanation

1.

$$z^2 + (\bar{z})^2 = 4 \Rightarrow (x+iy)^2 + (x-iy)^2 = 4$$

$$2(x^2 - y^2) = 4 \Rightarrow (x^2 - y^2) = 2$$ Hyperbola. **Ans: (b)**

2. Velocity of the rocket $v = \frac{c}{2}$. Using Lorentz transformation equations

$$x' = \gamma(x - vt) \;\; and\; t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad (1)$$

$$\because v = \frac{c}{2} \qquad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{c^2}{4c^2}}} = \frac{1}{\sqrt{\frac{3}{4}}} = \frac{2}{\sqrt{3}}$$

From (1) $x' = \frac{2}{\sqrt{3}}\left(x - \frac{ct}{2}\right) \;\; and\; t' = \frac{2}{\sqrt{3}}\left(t - \frac{x}{2c}\right)$ **Ans: (a)**

3. A classical ideal gas of N molecules in equilibrium is at temperature T. Each molecule has two energy levels, -ϵ and ϵ

The partition of each molecule is $z = e^{-\beta\varepsilon} + e^{\beta\varepsilon}$

Where $\beta = \frac{1}{kT}$

Mean energy $U = \frac{-\partial \ln z}{\partial \beta} \quad \Rightarrow U = -\frac{\partial}{\partial \beta} ln(e^{-\beta\varepsilon} + e^{\beta\varepsilon}) = \frac{\varepsilon e^{-\beta\varepsilon} - \varepsilon e^{\beta\varepsilon}}{e^{-\beta\varepsilon} + e^{\beta\varepsilon}}$

$$\Rightarrow U = \varepsilon\left[\frac{e^{-\beta\varepsilon} - e^{\beta\varepsilon}}{e^{-\beta\varepsilon} + e^{\beta\varepsilon}}\right] = -\varepsilon\left[\frac{e^{\beta\varepsilon} - e^{-\beta\varepsilon}}{e^{\beta\varepsilon} - e^{-\beta\varepsilon}}\right] = \varepsilon \tan\mathrm{h}\frac{\varepsilon}{kT}$$

For N particles $U = N\varepsilon \tan\frac{\varepsilon}{kT}$ **Ans: (c)**

4. Volume expansivity = $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$; Isothermal compressibility $k = \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_T$

Mathematical relation for Partial derivatives

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1 \quad \Rightarrow \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x = -\left(\frac{\partial x}{\partial z}\right)_y \qquad (1)$$

$$\frac{\beta}{k} = \frac{\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P}{-\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T} = \frac{-\left(\frac{\partial V}{\partial T}\right)_P}{\left(\frac{\partial V}{\partial P}\right)_T} = -\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial P}{\partial V}\right)_T = -\left(-\left(\frac{\partial P}{\partial T}\right)_V \right) \quad \text{Using (1)}$$

$$\Rightarrow \frac{\beta}{k} = \left(\frac{\partial P}{\partial T}\right)_V$$

Ans: (a)

5. Binary (Decimal)

$$\Rightarrow 1110101 - 0011110 = 1010111$$
$$(117) \quad - (30) \quad = (87)$$

Ans: (b)

6.

State	n_x	n_y	n_z	$n^2 = n_x{}^2 + n_y{}^2 + n_z^2$	degeneracy
Ground state	1	1	1	3	1
First Excited state	2	1	1	6	3
	1	2	1		
	1	1	2		
Second Excited state	2	2	1	9	3
	2	1	2		
	1	2	2		
Third Excited state	3	1	1	11	3
	1	3	1		
	1	1	3		
Fourth Excited state	2	2	2	12	1
Fifth Excited state	3	2	1	14	6
	3	1	2		
	2	1	3		
	2	3	1		
	1	2	3		
	1	3	2		

Degeneracy of the fifth excited state is six **Ans: (c)**

7. Particle of mass m and angular momentum L moves in space where its potential energy

$U(r) = k\,r^2 (k > 0)$ and r is the radical coordinate

force F $= -\frac{dU}{dr} = -2kr$

But, for circular orbit F $= \frac{m\,v^2}{r}$

Hence, $-2kr = \frac{mv^2}{r} \Rightarrow 2kr = \frac{(mvr)^2}{mr^3} = \frac{L^2}{mr^3}$

Where $L = mvr$, Leaving out the minus sign, $r^4 = \frac{L^2}{2mk} \Rightarrow r = \left[\frac{L^2}{2mk}\right]^{\frac{1}{4}}$ **Ans: (b)**

8. For a conservative field $\vec{F}$: $\vec{\nabla} \times \vec{F} = 0$

$$\vec{\nabla} \times \vec{F} = 0 = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (5x^2 + ay^2 + b\,xy) & (4x^2 + 4\,xy + y^2) & 0 \end{pmatrix}$$

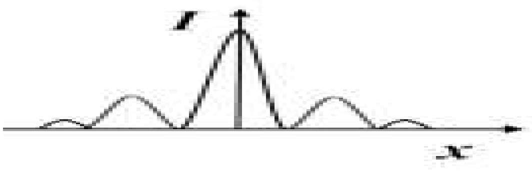
$$= \hat{x}(0) + \hat{y}(0) + \hat{z}(\{8x + 4y\} - \{2ay + bx\})$$

$$i.e., 0 = \hat{z}\left(\{8x+4y\} - \{2ay+bx\}\right) \Rightarrow (8-b)\,x + (4-2a)\,y = 0$$

$\Rightarrow \quad b = 8 \; and \; a = 2$ **Ans: (b)**

9. ALL (A), (B) and (C) are CORRECT statements for a conservative filed. But, the potential difference between ANY two points need NOT be zero. (The value of Zero is Applicable only in special cases wherein the two points lie on the same Equipotential line / surface) **Ans: (d)**

10.



Intensity distribution for the Fraunhofer diffraction due to a single slit. **Ans: (c)**

11. Using Taylor expansion $f(x) = e^{\sin x}$ around x=0

$$f(x) = f(0) + \frac{x}{1!}\, f^{'}(0) + \frac{x^2}{2!}\, f^{''}(0) + ... \qquad (1)$$

$$f(0) = e^{\sin 0} = e^{\circ} = 1$$

$$f^{'}(x) = e^{\sin x}\cos x \, ; = f^{'}(0) = e^{\sin 0}\cos 0 = 1$$

$$f^{''}(x) = e^{\sin x}\cos^2 x + e^{\sin x}(-\sin x) = e^0\,(1) + 0 = 1$$

From (1) $f(x)=1+x+\frac{x^2}{2}+.........................$ (2)

Given $f(x)=\sum_{n=0}^{\infty} a_n x^n = a_0+a_1 x+a_2 x^2+.........$ (3)

Comparing (2) and (3) $a_0=1,\ a_1=1,\ a_2=\frac{1}{2}$

$\therefore$ *The value of* $a_0+a_1+a_2=1+1+\frac{1}{2}=\frac{5}{2}$ **Ans: (c)**

12 I= $\oint\left[(\sin x-y)\ dx-(\sin y-x)dy\right]$ $C\rightarrow$ unit circle traversed in anticlock wise

Method 1

Using Green's theorem $\oint Mdx+Ndy=\iint_s\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dx\,dy$

M= sin x-y N = - (sin y – x) = x - sin y

$$\frac{\partial N}{\partial x}=1 \qquad \frac{\partial M}{\partial y}=-1$$

$$\therefore I=\iint_s\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dx\ dy=\iint_s 2\ dx\ dy=2\pi r^2$$

$$\therefore I=2\pi \quad [\because r=1]$$

Method 2

Using Stokes theorem $\oint \vec{F}.d\vec{l}=\iint(\vec{\nabla}\times\vec{F}).n\ ds$

Given the unit circle in the x-y plane. $\therefore n=k$

$$(\nabla\times F)=\begin{vmatrix}\hat{i} & j & k\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ \text{sin x-y} & \text{x-sin y} & 0\end{vmatrix}=k\left[1-(1)\right]=2k$$

$\therefore \iint_s(\vec{\nabla}\times\vec{F}).n\ ds=\iint_s 2\ k.\ k\,dx\,dy \ =2\pi r^2=2\pi(1)^2=2\pi$ **Ans: (b)**

13. The given problem is a standard book work in transient phenomena

The growth of current in RL-circuit is $i = i_0\left(1 - e^{-\frac{R}{L}t}\right)$

Here, $i_0 = \frac{\varepsilon}{R}$, the maximum steady current

$$i_0 = \frac{\varepsilon}{R} = \frac{1\ (V)}{1000\ (\Omega)} = 1\ mA$$

$$Hence, i = i_0\left(1 - e^{-\frac{R}{L}t}\right) = 1 \times \left(1 - e^{-\frac{10^3}{10^{-3}}\times 10^{-6}}\right) mA$$

$i = (1 - e^{-1})$ mA. **Ans: (c)**

14. An ideal gas in equilibrium at temperature T expands **isothermally** to twice its initial volume. If ΔS, ΔU and ΔF denote the changes in its entropy, internal energy and Helmholtz free energy respectively. **Since the expansion is isothermal and T is a constant** $\Delta T = 0$

Helmholtz free energy $F = U - TS$

$\therefore \Delta F = \Delta U - T\Delta S - S\Delta T$

$\therefore$ ΔU *and* ΔT are zero $\Delta F = -T\Delta S$

If $\Delta S > 0,\ then\, \Delta F < 0.$

Hence $\Delta S > 0,\ \Delta U = 0\ and\ \Delta F < 0$ **Ans: (b)**

15. In dilute gas, the number of molecules with free path lengths $\geq x$ is given by

$$N(x) = N_0\, e^{-x/\lambda}$$

N_0 is the total number of molecules and λ is the mean free path.

Total molecules $N_T = \int_0^\infty N_0\, e^{\frac{-x}{\lambda}}\, dx = N_0 \left[\frac{e^{\frac{-x}{\lambda}}}{\frac{-1}{\lambda}}\right]_0^\infty = N_0\lambda$ (1)

The fraction of molecules with free path lengths $\boldsymbol{\lambda}$ and 2 $\boldsymbol{\lambda}$ is

$$N = \int_{\lambda}^{2\lambda} N_0\, e^{-x/\lambda}\, dx = N_0 \left[\frac{e^{-x/\lambda}}{\frac{-1}{\lambda}} \right]_{\lambda}^{2\lambda}$$

$$\Rightarrow \quad N = -N_0\, \lambda \left[e^{-2} - e^{-1} \right] = N_0\, \lambda \left[e^{-1} - e^{-2} \right]$$

$$\Rightarrow \quad N = N_0\, \lambda \left[\frac{1}{e} - \frac{1}{e^2} \right] = N_0\, \lambda \left[\frac{e-1}{e^2} \right]$$

$$\Rightarrow \quad N = N_T \frac{(e-1)}{e^2} = \frac{N}{N_T} = \left[\frac{e-1}{e^2} \right]$$ **Ans: (d)**

16. **Deleted Question**

17. In plane polar coordinates, acceleration of a particle is

$$\vec{\boldsymbol{a}} = (\ddot{\boldsymbol{r}} - \boldsymbol{r}\dot{\boldsymbol{\theta}}^2)\hat{\boldsymbol{r}} + (\boldsymbol{r}\ddot{\boldsymbol{\theta}} + 2\dot{\boldsymbol{r}}\dot{\boldsymbol{\theta}})\hat{\boldsymbol{\theta}} \Rightarrow \vec{\boldsymbol{a}} = \boldsymbol{a_r}\hat{\boldsymbol{r}} + \boldsymbol{a_\theta}\hat{\boldsymbol{\theta}}$$ **(A standard result)**

$a_e = \ddot{r} - r\dot{\theta}^2$ is radial accn., and $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$ *is* tangential accn.

In our problem, $\dot{r} = const \Rightarrow \ddot{r} = 0$ *and* $\dot{\theta} = const, \Rightarrow \ddot{\theta} = 0$

$\therefore \quad \vec{\boldsymbol{a}} = -\boldsymbol{r}\dot{\boldsymbol{\theta}}^2\hat{\boldsymbol{r}} + 2\dot{\boldsymbol{r}}\dot{\boldsymbol{\theta}}\hat{\boldsymbol{\theta}}$ **Ans: (b)**

18. Let V *and* v *be the speeds of the planet at distance* R *and* r.

Then, angular momentum, $\mathrm{L} = \mathrm{mVR} = \mathrm{mvr}$ (1)

$$\Rightarrow v = \left(\frac{R}{r}\right) V \qquad (2)$$

Using law of conservation of energy, $-\frac{GMm}{r} + \frac{1}{2}\, mv^2 = -\frac{GMm}{R} + \frac{1}{2}\, mV^2$

$$\Rightarrow \frac{2GM}{r} - \frac{2GM}{R} = v^2 - V^2$$

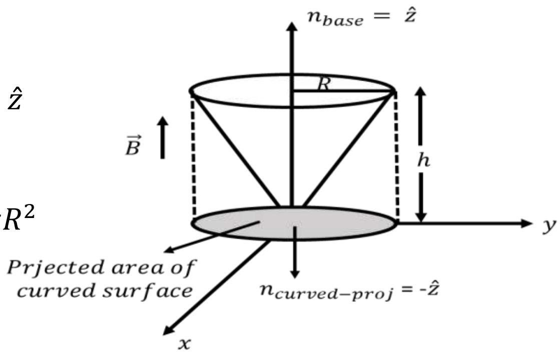
$$\Rightarrow \frac{2GM}{r} - \frac{2GM}{R} = \left[\left(\frac{R}{r}\right)V\right]^2 - V^2 = V^2\left(\frac{R^2 - r^2}{r^2}\right)$$ [using equ (2)]

$$\therefore V = \sqrt{\left(\frac{r^2}{R^2 - r^2}\right) 2GM \left(\frac{1}{r} - \frac{1}{R}\right)}$$

$$Angular\ momentum, L = mVR = mR\sqrt{\left(\frac{r^2}{R^2 - r^2}\right) 2GM \left(\frac{1}{r} - \frac{1}{R}\right)}$$

$$L = m\sqrt{\left(\frac{r^2}{R^2 - r^2}\right) \times R^2 \times 2GM \left(\frac{R-r}{rR}\right)} = \boldsymbol{m}\sqrt{\frac{\mathbf{2GM\,Rr}}{(\boldsymbol{R+r})}}$$ **Ans : (a)**

19.



Base Region: Magnetic flux, $\phi_b = \iint \vec{B}.\ \overrightarrow{dS} = \iint B_0\hat{z}\,.dS\,\hat{z}$

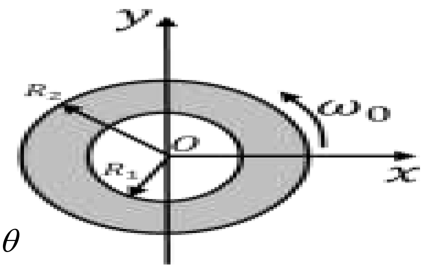
$$\phi_b = B_0 \iint dS = B_0\pi R^2$$

Curved surface: $\phi_c = \iint \vec{B}.\ \overrightarrow{dS} = \iint B_0\hat{z}\ .ds_{proj}(-\hat{z})$

$\phi_c = B_0\hat{z}\,.\pi R^2(-\hat{z}) = -B_0\pi R^2$ **Ans : (d)**

20. **Deleted Question**

A thin annular sheet, lying on the *xy*-plane, with R_1 *and* R_2 as its inner and outer radii, respectively. The sheet carries a uniform surface-charge density σ and spins about the origin O with a constant angular velocity $\vec{\omega} = \omega_0\hat{z}$.



Surface current in the annular sheet $k = \sigma v = \sigma r\omega_0$

Total current flow on the sheet $I = \int_0^{2\pi}\int_{R_1}^{R_2} k\,dA$ here $dA = r\,dr\,d\theta$

$$I = \int_0^{2\pi}\int_{R_1}^{R_2} (\sigma r\omega_0)(r\,d\,r\,d\,\theta) \quad = 2\pi\sigma\omega_0 \int_{R_1}^{R_2} r^2\,dr \ \Rightarrow\ 2\pi\sigma\omega_0\left[\frac{r^3}{3}\right]_{R_1}^{R_2} = 2\pi\sigma\omega_0\left[\frac{R_2^{\,3} - R_1^{\,3}}{3}\right]$$ **Ans :(a)**

21. Decay constant of the parent nucleus $\lambda_1 = \lambda$

Decay constant of the daughter nucleus $\lambda_2 = 10\lambda$

At t = 0 number of nuclei of the parent is $N_1 = N_0$ and there are no daughter nuclei i.e. $N_2 = 0$.

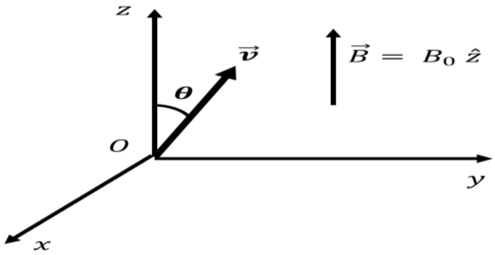
After a time t, number of nuclei of the parent $N_1 = N_0\,e^{-\lambda_1 t}$,

Number of nuclei of the daughter

$$N_2(t) = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right] = \frac{\lambda N_0}{10\lambda - \lambda}\left(e^{-\lambda t} - e^{-10\lambda t}\right) = \frac{\lambda N_0 e^{-\lambda t}}{9\lambda}\left[1 - e^{-9\lambda t}\right]$$

$$\therefore N_2(t) = \frac{1}{9}N_1(t)\left[1 - e^{-9\lambda t}\right] \quad \Rightarrow \frac{N_2(t)}{N_1(t)} = \frac{1}{9}\left[1 - e^{-9\lambda t}\right]$$ **Ans: (a)**

22. A uniform magnetic field $\vec{B} = B_0 z$, where $B_0 > 0$ exists as shown in the figure. A charged particle of mass *m* and charge *q* (*q* > *0*) is released at the origin, in the *yz*-plane, with a velocity $\vec{v}$ directed at an angle $\theta = 45^\circ$ with respect to the positive z-axis.



From fig, $\vec{v} = v\cos 45^o\,\hat{z} + v\sin 45^o\,\hat{y} = \frac{v}{\sqrt{2}}\hat{z} + \frac{v}{\sqrt{2}}\hat{y}$ and given $\vec{B} = B_0\hat{z}$

Magnetic force on charge q is $\vec{F}_{mag} = q\left(\vec{v} \times \vec{B}\right)$

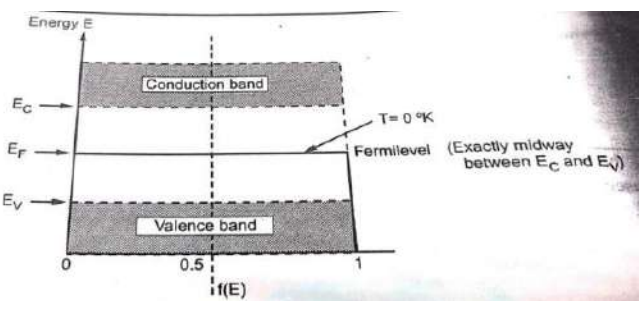
$$= q\begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & v/\sqrt{2} & v/\sqrt{2} \\ 0 & 0 & B_0 \end{pmatrix} = \hat{x}\left(B_0\frac{v}{\sqrt{2}}\right) q$$

Hence the acceleration is $\vec{\boldsymbol{a}} = \vec{\boldsymbol{F}}_{mag}/\boldsymbol{m} = \left[B_0 vq/\sqrt{2}\,m\right]x$ **option(a) is correct**

Note - Particle describes a helical path with z-axis as the axis of the helix.

Hence **options (C) and (D) are Incorrect** **Ans: (a)**

23.



E_C : Be the lowest energy level in the conduction band

E_F : Indicates the probability of occupancy of an energy level by an electron

E_V : Be the highest energy level in the valence band **Ans: (c)**

24. $\vec{B}(\rho\,,\varphi\,,z\,,t) = k\,\rho^3 t^3 \hat{z}$ ($given$)(Cylindrical coordinates are used)

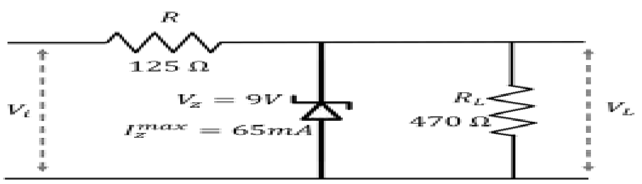
The magnetic flux, $\phi_m = \iint \vec{B}.\,\overrightarrow{dS} = \iint k\,\rho^3 t^3 \hat{z}\ .(\rho\,d\rho\,d\varphi)\hat{z}$

$$\phi_m = k\,t^3 \int \rho^4\,d\rho \times \int d\varphi = k\,t^3 \times \frac{R^5}{5} \times 2\pi$$

Hence, the induced emf, $\varepsilon = -\frac{d\varphi}{dt} = -2\pi k \times 3\,t^2 \times \frac{R^5}{5}$

Ignoring -ve sign, emf induced is $\boldsymbol{\varepsilon = \frac{6}{5}\,\pi\,k\,t^2 R^5}$ **Ans: (a)**

25.



Given Zenes dioide will be always 'ON' state and V_z

$\therefore V_i^{\min} > V_z \quad \therefore V_i^{\min} = 11.4V$ **(From options (c) and (d))**

When V_z Is on, the current through R $= \dfrac{V_i^{\max} - V_z}{R} = \dfrac{V_i^{\max} - 9}{125}$ **Options (A) and (B) are wrong**.

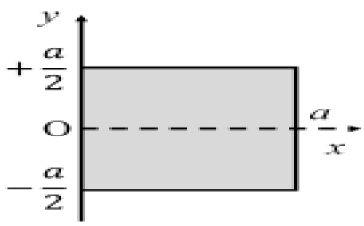
But current through R= $I_z^{\max} + I_L$

$$\therefore \frac{V_i^{\max} - 9}{125} = 65\,m.A + \frac{9}{470} = 65\,m\,A + 19.14\,m.A = 84.14\times10^{-3}$$

$$V_i^{\max} - 9 = 84.14\times125\times10^{-3} = 10517.5\times10^{-3}\ \therefore V_i^{\max} = 19.5V$$

$$\therefore V_i^{\min} = 11.4V \text{ and } V_i^{\max} = 19.5V$$

26.



Consider a rectangular strip of width dx(at distance x) and of height a (parallel to $y - axis$)

The mass of this strip is $dM = \sigma\,(x) \times a\,dx = \sigma_0\left[1 - \frac{x}{a}\right] \times a\,dx$

Hence total mass, $M = \int dM = \sigma_0\,a \int_0^a \left[1 - \frac{a}{x}\right] dx = \sigma_0\,a \times \left(a - \frac{a^2}{2a}\right)$

Total mass, $M = \frac{1}{2}\sigma_0 a^2$ (1)

Now, the M.I of this strip about y-axis ,$dI = dM \times a^2 = \sigma_0 \left[1 - \frac{x}{a}\right] \times a\,dx \times x^2$

Hence the M.I of the whole sheet about y-axis, $I = \int_0^a \sigma_0\, a \times \left[1 - \frac{x}{a}\right] \times x^2\, dx$

$$I = \sigma_0 a \int_0^a \left[x^2 - \frac{x^3}{a}\right] dx = \sigma_0 a \times \left[\left[\frac{x^3}{3}\right]_0^a - \left[\frac{x^4}{4a}\right]_0^a\right]$$

$$I = \sigma_0 a \left[\frac{a^3}{3} - \frac{a^4}{4a}\right] = \sigma_0 a \times \frac{a^3}{12} = \sigma_0 \frac{a^4}{12}$$

$$I = \sigma_0 \frac{a^4}{12} = \sigma_0 \frac{a^3}{2} \times a^2 \times \frac{1}{6} = \boldsymbol{\frac{M\,a^2}{6}}\ (using\ eqn. (1))$$ **Ans: (c)**

27. $y = 2a\sin\omega t \Rightarrow \frac{y^2}{4a^2} = \sin^2\omega t$.............. (1)

$\left[1 - \frac{y^2}{4a^2}\right] = 1 - \sin^2\omega t = \cos^2\omega t$............. (2)

$x = a\sin(2\omega t + \pi) = -a\sin\ 2\omega t = -a(2\sin\omega t\cos\omega t)$

$x^2 = a^2 \times 4\sin^2\omega t\cos^2\omega t$

$\Rightarrow \quad x^2 = a^2 \times 4 \times \frac{y^2}{4a^2}\left[\frac{1-y^2}{4a^2}\right] = 1 \Rightarrow x^2 = y^2\left[\frac{1-y^2}{4a^2}\right]$ **Ans: (d)**

28.

P is proportional to T^2

$U = P\,V$ (1) $\qquad P\,\alpha\,T^2 \Rightarrow P\,a\,T^2$ where a is a constant (2)

$\therefore U = aT^2V$ (3) $\qquad$ differentiating equ (3) $\quad dU = 2a(T\,dT)V + aT^2\,dV$ (4)

$\because T\,dS = dU + PdV$ (5)

$\Rightarrow dS = \frac{dU + PdV}{T} = \frac{2aTV\,dT + a\,T^2\ dV + aT^2\,dV}{T}$ using (4) and (3)

$\Rightarrow dS = 2a\ Vd\ T + 2aT\,dV = 2a\,d(VT)$

Integrating bothsides $S = 2aVT + a$ constant (6)

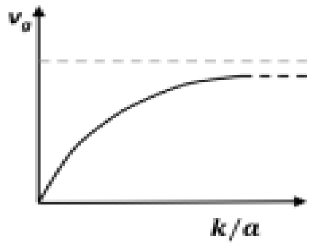
from (6) $\ S\ \alpha\ VT$ or $S\ \alpha\ V\sqrt{P}$ $\left[\because P\,\alpha\,T^2 \text{ from (2)}\right]$

$$\Rightarrow S\,\alpha\sqrt{PV^2} \;\Rightarrow\; S\,\alpha\sqrt{(PV)V} \Rightarrow S\,\alpha\sqrt{UV} \quad \text{using}\,(1)$$ **Ans: (d)**

29. The dispersion relation for certain type of waves is

$$\omega = \sqrt{k^2 + a^2} \ldots\ldots\ldots\ldots\ldots(1)$$

Where k is the wave vector and a is a constant.



From (1) $\omega^2 = k^2 + a^2$

differentiating $2\omega \dfrac{\partial\omega}{\partial k} = 2k \;\Rightarrow\; \dfrac{\partial\omega}{\partial k} = \dfrac{k}{\omega} = \dfrac{k}{\sqrt{k^2+a^2}}$

$\therefore$ group velocity $v_g = \dfrac{\partial\omega}{\partial k} = \dfrac{k}{\sqrt{k^2+a^2}} = \Rightarrow v_g = \dfrac{1}{\sqrt{1+\left(\dfrac{a}{k}\right)^2}}$

When $\dfrac{k}{a} = 0, \;\; \dfrac{a}{k} = \infty, \;\; v_g = 0$ When $k \to \infty$ $v_g = a$ constant **Ans : (b)**

30. Given a binary number with m digits, where m is an even number. The binary number has alternating $1's\, and\, 0's$, with digit 1 in the highest place value.

		Binary number	decimal equivalent
When	m=2	10	2
	m=4	1010	10
	m=6	101010	42

From option (D) decimal equivalent= $= \dfrac{2}{3}(2^m - 1)$

check $m = 2 \;\Rightarrow \dfrac{2}{3}(2^1 - 1) = 2$

$m = 4 \;\Rightarrow \dfrac{2}{3}(2^4 - 1) = 10$

$m = 6 \;\Rightarrow \dfrac{2}{3}(2^6 - 1) = 42$ **Ans: (d)**

Section B

31. $M = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix}, where\, a, b > 0$

a) $M^T = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix} = M \quad \therefore M\ is\ a\ real\ symmetric\ matrix$ **Option (a) is correct**.

$d)\ \ Product\ of\ \ eigenvalues = |M| \Rightarrow \lambda_1 \lambda_2 = -a^2.$ **Option (d) is wrong**

c) $\because \lambda_1 \lambda_2 = -a^2$ one of the eigen values must be negative **Option (c) is correct**

b) sum of the eigen values $\because \lambda_1 + \lambda_2 =$ trace of the matrix

$$\because \lambda_1 + \lambda_2 = b \ \ \Rightarrow \ \lambda_1 = b - \lambda_2$$

If λ_2 is negative, then $\lambda_1 > b$. **Option (b) is correct** **Ans: (a, b, c)**

32. In the Compton scattering of electrons, by photons incident with wavelength λ,

the scattered wavelength $\lambda^{'} = \lambda + \lambda_c (1 - \cos\varphi)$

$where\ \lambda_c = \dfrac{h}{m_0 c}$ is Compton wave length.

$$\therefore \lambda^{'} - \lambda = \lambda_c (1 - \cos\varphi) \Rightarrow \frac{\lambda^{'} - \lambda}{\lambda} = \frac{\lambda_c}{\lambda} (1 - \cos\varphi)$$

$$\Rightarrow \frac{\Delta\lambda}{\lambda} = \frac{\lambda_c}{\lambda}(1 - \cos\varphi) \qquad (1)$$

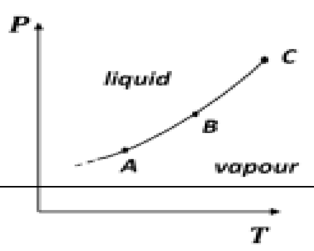
From Equation (1) $\dfrac{\Delta\lambda}{\lambda}$ depends on λ **Option (a) is wrong**

$\dfrac{\Delta\lambda}{\lambda}$ increasing with decreasing λ **Option (b) is correct**

$\dfrac{\Delta\lambda}{\lambda}\ \alpha\, (1 - \cos\varphi)$ *i.e.*, $\dfrac{\Delta\lambda}{\lambda}$ increases **with increasing angle of deflection of the photon**

Option (d) is correct, but (c) is wrong **Ans: (b,d)**

33.



Figure

Between vapour phase and liquid phase, the chemical potential must be same

i.e., $\mu_1 = \mu_2$ along AB **option (a) is correct**.

Specific volume changes from liquid phase to vapour phase. ∴ $v_1 \neq v_2$ along AB

option (b) is wrong.

Entropy will be discontinuous change from liquid phase to vapour phase. option (c) is wrong

But at the critical point, these phases are constant and volume does not change

i.e., $v_1 = v_2$ at the point C. **Option (d) is correct.** **Ans : (a,d)**

34. A particle is executing simple harmonic motion with time period T. x, v and a denote the displacement, velocity and acceleration of the particle respectively, at time t.

(a) $\frac{aT}{x} = -\frac{\omega^2 xT}{x}$ $\left[\because \text{ acceleration } a = -\omega^2 x\right] \Rightarrow \omega^2 T$ constant. **It does not change with time.**

option (a) is correct

(b) $\left(aT - 2\pi f\right) = \left(aT + \frac{2\pi}{T}\right)$ **depends on t, since acceleration varies with time t.**

option (b) is wrong

(c) $x = A\sin\omega t \quad and \quad v = A\omega\cos\omega t.$ **x and v are not related by a straight line.**

option (c) is wrong.

(d) $v = A\omega\cos\omega t$ and $a = -\omega^2 A\sin\omega t$

$$\therefore \left(\frac{v}{A\omega}\right)^2 + \left(\frac{a}{-\omega^2 A}\right)^2 = \cos^2\omega t + \sin^2\omega t \Rightarrow \left(\frac{v}{A\omega}\right)^2 + \left(\frac{a}{-\omega^2 A}\right)^2 = 1$$ **Equation of an ellipse.**

Option (d) is correct. **Ans: (a, d)**

35. A linearly polarized light beam travels from origin to point A (1,0,0). At the point A, the light is reflected by a mirror towards point B $(1, -1, 0)$. A second mirror located at point B then reflects the

light towards point C $(1, -1, 1)$. Let $n(x, y, z)$ represent the direction of polarization of light at (x, y, z).

(a) Given $n(0,0,0)\,y$ **ie polarized light travels along** y **direction. At** $A(1,0,0)$

it is reflected by the mirror towards to the point $B(1,-1,0)$ **i .e., it travels along - z direction.**

The second mirror reflects the light towards the point $C(1,-1,1)$.

Now the light travels along – z to x direction.

$\therefore$ direction $y \to -\hat{z} \to x$ **option (a) is correct**

(b) in this case, the direction of polarized light $\hat{z} \to -x \to y$ **option (b) is correct**

(c) and (d) are wrong **Ans: (a,b)**

36. Given (r,θ) denote the polar coordinates of a particle moving in a plane.

In plane polar coordinates, $\vec{r} = r\cos\theta\,\hat{\imath} + r\sin\theta\,\hat{\jmath}$

$$\hat{r} = \frac{\partial\vec{r}/\partial r}{\left|\partial\vec{r}/\partial r\right|} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath} \qquad (1)$$

$$\hat{\theta} = \frac{\partial\vec{r}/\partial\theta}{\left|\partial\vec{r}/\partial\theta\right|} = \frac{-r\sin\theta\,\hat{\imath} + r\ \sin\theta\,\hat{\jmath}}{r} = -\sin\theta\,\hat{\imath} + \ \sin\theta\,\hat{\jmath} \qquad (2)$$

From (1) $\frac{\partial\hat{r}}{\partial\theta} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath} = \ \hat{\theta}$ **option (a) is correct**

From (1) $\frac{\partial\hat{r}}{\partial r} = \ 0$ **option (b) is wrong**

From (2) $\frac{\partial\hat{\theta}}{\partial\theta} = \ -\cos\theta\,\hat{\imath} - \sin\theta\,\hat{\jmath} = \ \hat{\theta} = \ -\hat{r}$ **option (c) is correct**

From (2) $\frac{\partial\hat{\theta}}{\partial r} = 0$ **option (d) is wrong** **Ans : (a,c)**

37. The electric field associated with an electromagnetic radiation is given by

$$E = a\left(1+\cos\omega_1 t\right)\cos\omega_2 t = a\cos\omega_2 t + a\cos\omega_1 t\ \cos\omega_2 t$$

$$= a\cos\omega_2 t + \frac{a}{2}\left[\cos\left(\omega_1+\omega_2\right)t + \cos\left(\omega_1-\omega_2\right)t\right]\ \therefore \text{Frequencies present are } \omega_2, \omega_1+\omega_2 \text{ and } \omega_1-\omega_2$$

Ans: (b,c,d)

38. A string of length L is stretched between two points x = 0 and x = L and the end points are rigidly clamped. **Locations x = 0 and x = L are NODES. That is their displacements are always Zero. ALL options satisfy the location x = 0**

However, at x = L , only (b), (c) and (d) alone satisfy. **Ans: (b,c,d)**

(39)

$$Y = \overline{PQ}\,R + Q\overline{R} + \overline{P}\,QR + PQR\ = (\overline{P}+\overline{Q})\mathrm{R} + Q\overline{R} + QR \quad [\because \overline{P}+P=1]$$

$$= \overline{P}\,R + \overline{Q}\ R + Q\,\overline{R} + QR\ = \overline{P}\,R + (\overline{Q}+Q)\,R + Q\,\overline{R}$$

$$= \overline{P}\,R + R + Q\,\overline{R} \quad \Rightarrow \left(\overline{P}+1\right)R + Q\overline{R} = R + Q\,\overline{R} = R + Q$$

Ans: (d)

40.

41. $I = \iint (x^2+y^2)\ dx\,dy$ Given radius of the disc is 2.

Using spherical polar coordinates

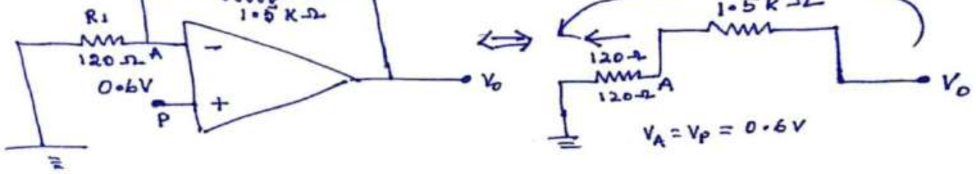
$x = r\cos\theta$ $\quad y = r\sin\theta$ $\quad dx\,dy = r\,dr\,d\theta.$

$r \to 0\ to\ 2$ $\quad \theta \to 0\ to\ 2\pi$

$$I = \int_{r=o}^{2}\int_{\theta=0}^{2\pi} r^2(r\,dr d\theta) \quad = 2\pi\int_0^2 r^3\ dr = 2\pi\left(\frac{r^4}{4}\right)^2 = 8\pi$$

Ans: (8)

42.



$$\frac{V_0}{120\ \Omega + 1.5 k\Omega} = \frac{0.6}{120\ \Omega} = I_0$$

Output current $I_o = \frac{0.6}{120} = \frac{6}{1200} = \frac{1}{200} = \frac{5}{1000} amp = 5 mA.$ **Ans: (5)**

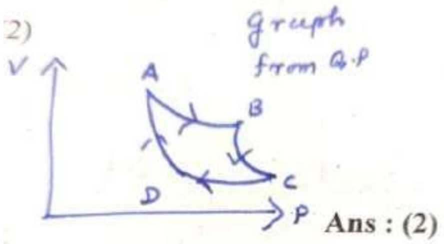
43. V_A,V_B,V_C and V_D are the volumes of the gas at A,B,C,D respectively.

Given $\frac{V_C}{V_B} = 2$; $\frac{V_D}{V_A} = ?$

AD and BC represents adiabatic paths. For adiabatic process $TV^{\gamma-1}$ = constant

For BC $T_1 V_B^{\gamma-1} = T_2 V_C^{\gamma-1}$ (1)

For AD $T_1 V_A^{\gamma-1} = T_2 V_D^{\gamma-1}$ (2)



$$(1) \div (2) \left(\frac{V_B}{V_A}\right)^{\gamma-1} = \left(\frac{V_C}{V_D}\right)^{\gamma-1} \Rightarrow \frac{V_B}{V_A} = \frac{V_C}{V_D}$$

$$\frac{V_C}{V_B} = \frac{V_D}{V_A} \therefore \frac{V_C}{V_B} = 2, \quad \frac{V_D}{V_A} = 2$$ **Ans: (2)**

44. The relation between the eccentricity ϵ and the apogee distance R_1 and perigee distance

$R_2\,(< R_1)$ is $\frac{1+\epsilon}{1-\epsilon} = \frac{R_1}{R_2}$ (Or) $\epsilon = \frac{R_1 - R_2}{R_1 + R_2}$

In the given problem, $R_1 = R_E + Height\ of\ apogee\ distance$

$$R_1 = 6500 + 4500 = 11{,}000\ km$$

And, $R_2 = R_E + Height\ of\ perigee\ distance$

$$R_1 = 6500 + 2500 = 9{,}000\ km$$

$$\therefore\ \epsilon = \frac{11000 - 9000}{11000 + 9000} = \frac{2000}{20000} = 0.1$$

Eccentricity, $\boldsymbol{\epsilon = 0.1}$ **Ans: (0.1)**

45. Three masses $m_1 = 1,\ m_2 = 2\ and\ m_3 = 3$ are located on the x-axis such that their center of mass

is at x = 1 $x_{CM} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots} \Rightarrow 1 = \frac{(1\times x_1) + (2\times x_2) + (3\times x_3)}{1+2+3}$

$$\Rightarrow \quad 6 = x_1 + 2\,x_2 + 3\,x_3 \qquad (1)$$

After adding mass $m_4 = 4$ which is placed at x = 0, the new centre of mass is 3

$$\therefore\ 3 = \frac{x_1 + 2\,x_2 + 3\,x_3 + 4\,x_0}{10} \quad \Longrightarrow \quad 30 = x_1 + 2\,x_2 + 3\,x_3 + 4\,x_0 \qquad (2)$$

From eqns. (1) and (2) we get, $24 = 4x_0 \quad \Longrightarrow \quad \boldsymbol{x_0 = 6}$ **Ans :(b)**

46. Separation distance between the two objects, $d = 0.35\ m$

Distance of objects from the eye, $D = 1000\ m$

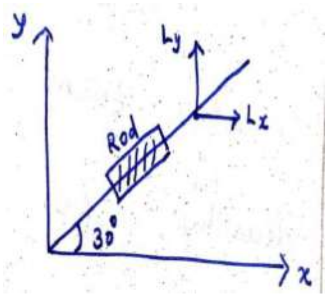
Angular separation at eye (angle of resolution) = $\frac{d}{D} = \frac{0.35}{1000}\ radian$

Angle of resolution = $\frac{0.35}{1000} \times \frac{180}{\pi} \times 60 \times 60\ seconds\ = 72.23$

The angular resolution of eye ≈ 72 **Ans : (72)**

47. **Proper length of the rod L_0 = 3m**

Given the rod moves with a velocity $\frac{c}{2}$, making an angle of 30° with respect to x – axis.



x component of the rod $L_x = L_0\sqrt{1 - \frac{v^2}{c^2}}\cos 30^\circ$

$$L_x = L_0\sqrt{1 - \frac{c^2}{4c^2}}\frac{\sqrt{3}}{2} = L_0\frac{3}{4}$$

y component of the rod $L_y = L_0 \sin 30^\circ = \frac{L_0}{2}$

$$\therefore\ L = \sqrt{L_x^2 + L_y^2} = \sqrt{\left(\frac{3L_0}{4}\right)^2 + \left(\frac{L_0}{2}\right)^2} = L_0\sqrt{\frac{9}{16} + \frac{1}{4}} = \frac{\sqrt{13}}{4}L_0$$

∴ The change in length due to Lorentz contraction

$$L = L_0 - \frac{\sqrt{13}}{4}L_0 = L_0\left(1 - \frac{\sqrt{13}}{4}\right) = (3m) \times 0.1 = 0.3m$$ **Ans: (0.3 m)**

48. Speed of an electron of hydrogen atom in the nth Bohr orbit $V_n = \dfrac{Z e^2}{2\varepsilon_0 \times n h}$

Speed of the electron on the 2nd orbit of Hydrogen atom $= \dfrac{e^2}{2\varepsilon_0 \times 2h}$

$$= \frac{(1.6\times10^{-19})^2}{4\times8.854\times10^{-12}\times6.63\times10^{-34}} = \frac{2.56\times10^{-38}}{4\times8.854\times6.63\times10^{-46}} = 1.09\times10^{6}\ m/s$$ **Ans : (1.09)**

49. Given unit circle C in the xy plane with center at the origin

Given $\vec{F}(x,y,x) = -2y\,x - 3z\,y + x\,z$

Using Stoke's theorem $\oint \vec{F}.d\vec{l} = \iint_s (\vec{\nabla}\times\vec{F})\,.n\,ds$

$$\left(\vec{\nabla}\times\vec{F}\right) = \begin{vmatrix} \hat{i} & j & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ -2y & -3z & x \end{vmatrix} = \hat{i}(0+3) - j(1-0) + k(0+2) = 3\hat{i} - j + 2k$$

$$\therefore \iint_s (\vec{\nabla}\times\vec{F}).n\ ds = \iint_s (3\hat{i} - j + 2\ k)\ k\,ds = 2\iint_s ds = 2\pi r^2 = 2\pi(1)^2 = 2\pi$$ **Ans: (2)**

50.

51. **Given differential equation** $y'' + 4y' + 5y = 0$ **(1)**

C.F is $\left(m^2 + 4m + 5\right) = 0 \Rightarrow m = \dfrac{-4 \pm \sqrt{4^2 - 4(5)(1)}}{2} = \dfrac{-4 \pm 2i}{2} = -2 \pm i$

$\therefore$ **The solution of the equation (1)** $y = e^{-2x}\left(A\cos x + B\sin x\right)$, where A and B are constants

Given $y(0) = 0 \quad \therefore \quad A = 0 \Rightarrow y = Be^{-2x}\sin x$ **(2)**

Differentiating equation (1)

$$y' = -2Be^{-2x}\sin x + Be^{-2x}\cos x.$$

Given $y'(0) = 1 \quad \therefore \quad 1 = 0 + B \quad \Rightarrow \quad B = 1$

$$y = e^{-2x}\sin x \Rightarrow y\left(\frac{\pi}{2}\right) = e^{-2\frac{\pi}{2}}\sin\frac{\pi}{2} \quad \therefore \quad y = e^{-\pi} = 0.043$$ **Ans: (0.043)**

52. An atom of a monoatomic gas has 3 translational degrees of freedom and its kinetic energy is

$K.E = \frac{1}{2}\, M\, {v_{rms}}^2 = 3\ \times \frac{1}{2} k_B\, T \quad \therefore\ v_{rms} = \sqrt{\frac{3\, k_B\, T}{M}}$

Hence in our problem, $v_{1_{rms}} = \sqrt{\frac{3\, k_B\, T}{m}}\ and\ v_{2_{rms}} = \sqrt{\frac{3\, k_B\, T}{2m}}$

Method 1:

To take the average of v_{rms}, we take the average of ${v_{rms}}^2$

$$\text{average value of } v^2_{rms} = \frac{1}{2} \times \left[{v^2}_{1_{rms}} + {v^2}_{2_{rms}} \right] = \frac{1}{2} \times \left(\frac{3k_B\, T}{m} \right) \times \left(1 + \frac{1}{2} \right)$$

$${v^2}_{av_{rms}} = \frac{3}{4} \times \left(\frac{3k_B\, T}{m} \right)$$

Hence, $v_{average_{rms}} = \frac{3}{2} \sqrt{\frac{k_B\, T}{m}}$

Comparing this with the given answer, $v_{rms} = x \sqrt{\frac{k_B\, T}{m}}$, we get $\boldsymbol{x = 1.5}$(Answer)

Method 2:

$$\text{average value of } v_{\text{rms}} = \frac{1}{2} \times \left[v_{1_{rms}} + v_{2_{rms}} \right] = \frac{1}{2} \times \left[\sqrt{\frac{3\, k_B\, T}{m}} + \sqrt{\frac{3\, k_B\, T}{2m}} \right]$$

$$v_{average_{rms}} = \frac{1}{2} \times \left[\sqrt{3} + \sqrt{\frac{3}{2}} \right] \times \left[\sqrt{\frac{k_B\, T}{m}} \right]$$

$$v_{ave_{rms}} = \frac{1}{2} \times \left[1.732 + \frac{1.782}{1.414} \right] \times \sqrt{\frac{k_B\, T}{m}}$$

$$v_{ave_{rms}} = \frac{1}{2} \times \left[1.732 + 1.225 \right] \times \sqrt{\frac{k_B\, T}{m}}$$

$$v_{ave_{rms}} = 1.4785 \times \sqrt{\frac{k_B\, T}{m}}$$

$$v_{ave_{rms}} \approx 1.49 \times \sqrt{\frac{k_B\,T}{m}}$$

Comparing this with the given answer, $v_{rms} = x\sqrt{\frac{k_B\,T}{m}}$, we get $\boldsymbol{x = 1.49}$ **Ans: (1.49)**

53. **A hot body with constant capacity 800 J/K at a temperature 925 K is dropped gently into a vessel containing 1 kg of water at temperature 300 K and combined system is allowed to reach the equilibrium**.

Heat lost by the hot body = heat gained by the water.

$$800\,(925 - T_f) = 4200\,(T_f - 300)$$

Where T_f is the final temperature of the combined system.

$$(925 - T_f) = 5.25\,(T_f - 300) \Rightarrow T_f = \frac{2500}{6.25} = 400\,K$$

The change in entropy $\Delta S = \frac{dQ}{T} = C\frac{dT}{T}$ where C is the heat capacity.

$$\text{For hot body } \Delta S_H = C\int_{T_i}^{T_f}\frac{dT}{T} = C\ln\left(\frac{T_f}{T_i}\right)$$

$$= C\ln\left(\frac{400}{925}\right) \Rightarrow \Delta S_H = C\ln(0.432) = -800\times 0.839 = -670.66\ \ J/K$$

For water $\Delta S_W = ms\frac{dT}{T} = ms\int_{T_i}^{T_f}\frac{dT}{T} = ms\ln\left(\frac{T_f}{T_i}\right)$

$$\Rightarrow\ \Delta S_W = 1\times 4200\ln\left(\frac{400}{300}\right) = 4200\times 0.287 = 1208.26\ \ J/K$$

$\therefore$ The change in the total entropy $\Delta S = \Delta S_H + \Delta S_W = -670.66 + 1208.26 = 537.6\ \ J/K$

Ans: (537.6)

54. Momentum of the electron in the region I $p_1 = \sqrt{2mE}$

Momentum of the electron in the region II $p_2 = \sqrt{2m(E-U_0)}$

$$R = \left(\frac{p_1 - p_2}{p_1 + p_2}\right)^2 = \left(\frac{\sqrt{2mE} - \sqrt{2m(E-U_0)}}{\sqrt{2mE} + \sqrt{2m(E-U_0)}}\right)^2 \qquad (1)$$

Given in the limit $E \gg U_0, R = \dfrac{U_0^2}{nE^2}$ (2)

From (1) and (2)

$$\left(\frac{1-\sqrt{1-\frac{U_0}{E}}}{1+\sqrt{1-\frac{U_0}{E}}}\right)^2 = \frac{U_0^2}{nE^2}$$

Taking square root on both sides

$$\frac{1-\left(1-\frac{U_0}{E}\right)^{1/2}}{1+\left(1-\frac{U_0}{E}\right)^{1/2}} = \frac{U_0}{\sqrt{n}E} \Rightarrow \frac{1-\left(1-\frac{U_0}{2E}\right)}{1+\left(1-\frac{U_0}{2E}\right)} = \frac{U_0}{\sqrt{n}E} \quad \because E \gg U_0 \qquad (3)$$

Using Binomial approximation $(1\pm x)^n \approx 1 \pm nx$

$$\frac{\frac{1}{2}\frac{U_0}{E}}{2} = \frac{U_0}{\sqrt{n}E} \Rightarrow \frac{1}{4} = \frac{1}{\sqrt{n}} \Rightarrow n = 16$$ (**Ans: 16**)

55. **Current density for a fluid flow** $\vec{J}(x,y,z) = \dfrac{8e^t}{(1+x^2+y^2+z^2)}x$ **(1)**

Continuity equation $\vec{\nabla}.\vec{J} + \dfrac{\partial\rho}{\partial t} = 0$ **(2)**

$$\vec{\nabla}.\vec{J} = 8e^t\left[\frac{-2x}{(1+x^2+y^2+z^2)^2}\right] = \frac{16\,x\,e^t}{(1+x^2+y^2+z^2)^2}$$

From (2) $\dfrac{\partial\rho}{\partial t} = -\vec{\nabla}.\vec{J} = \dfrac{-16\,x\,e^t}{(1+x^2+y^2+z^2)^2}$

Integrating the above equation

$$\rho = \frac{16\,x}{(1+x^2+y^2+z^2)^2}\int e^t dt + C \qquad \rho(x,y,z,t) = \frac{16\,x\,e^t}{(1+x^2+y^2+z^2)^2} + C \qquad \textbf{(3)}$$

Given at time t = 0, ρ(x,y,z,t) = 1

$$\therefore 1 = \frac{16\,x\,e^t}{(1+x^2+y^2+z^2)^2} + C \quad \Rightarrow \quad C = 1 - \frac{16\,x\,e^t}{(1+x^2+y^2+z^2)^2}$$

From (3) $\rho(x,y,z,t)=\dfrac{16\,x\,e^{t}}{\left(1+x^2+y^2+z^2\right)^2}+1-\dfrac{16\,x\,e^{t}}{\left(1+x^2+y^2+z^2\right)^2}$

$\rho(1,1,1,1)=\dfrac{16\times1\times e^{1}}{(4)^2}+1-\dfrac{16}{(4)^2}=e \qquad \rho(1,1,1,1)=e=2.71$ **Ans: (2.71)**

56. Given: $q=-5\ \mu C.\ \vec{E}=\left(8r\sin\theta\,\hat{r}+4r\cos\theta\,\hat{\theta}\right),\ A\,(r,\theta)=\left(10,\frac{\pi}{6}\right), B\,(r,\theta)=\left(10,\frac{\pi}{2}\right)$

Work done in moving charge q between the two points is

$$W=q\left[\int_A^B \vec{E}(r,\theta)\,.\,\overrightarrow{dr}\right]=q\int_A^B (E_r\,dr+\ E_\theta\,r\,d\theta\,)=q\int_A^B E_\theta\,r\,d\theta$$

(Since there is NO change in r coordinates)

$$W=q\int_{\pi/6}^{\pi/2}(4r\cos\theta\,).\,r\,d\theta$$

$$W=q\ \times 4\,r^2\int_{\pi/6}^{\pi/2}\cos\theta\,d\theta$$

$$=5\times10^{-6}\times4\times10^2\times[\sin\theta]_{\pi/6}^{\pi/2}$$

$$=20\ \times\ 10^{-4}\ \times\ \left(1-\tfrac{1}{2}\right)=0.1\text{ mJ}$$ **Ans: (0.1 mJ)**

57. **Deleted Question**

58. C = 30 ° i_p =?

$\mu=\tan i_p=\dfrac{i}{\sin C}\ \Rightarrow\tan i_p=\dfrac{1}{\sin 30^\circ}=2\Rightarrow\ i_p=63^\circ.$ **Ans:** (63°)

59. Given that $R=150\ \Omega\,, L=0.2\ H\ C=30\ \mu F.$

$$V=220\ V\,, f=50\ Hz.$$

$$\omega=2\pi\,f=100\,f$$

$$P_{loss}=\ I^2R=\left(\frac{V}{Z}\right)^2R=\frac{V^2\,R}{Z^2}$$

To determine Z

$$\left(L\omega - \frac{1}{C\omega}\right) = \left(20\,\pi - \frac{1000}{3\,\pi}\right) = 62.857 - 106.06 = 43.21$$

$$Z^2 = R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2 = 150^2 + 43.21^2 = 24{,}367$$

Hence, $P_{loss} = \frac{V^2\,R}{Z^2} = \frac{220^2 \times 150}{24{,}367} = 297.94\,W \quad = \quad \boldsymbol{P_{loss}} \approx \mathbf{298\,W}$ **Ans: (298 W)**

60. ***Inside dielectric medium***, $\iint \vec{D}\,.\overrightarrow{dS} = Q_{free}$ & the electric field $E = {}^{D}/_{\epsilon_r \epsilon_0}$

Energy of electric filed inside dielectric is $U = \frac{1}{2}\iiint \vec{D}\,.\vec{E}\ d\tau$

In free region, (outside the sphere) , $\iint \vec{E}\,.\overrightarrow{dS} = {}^{Q_{total}}/_{\epsilon_0}$; $E = {}^{D}/_{\epsilon_0}$&

the energy is $U = \frac{1}{2}\epsilon_0 \iiint E^2\ d\tau$

For $0 < r \le a$(point inside sphere)

$$\iint \vec{D}\,.\overrightarrow{dS} = Q_{free} \rightarrow \iint \vec{D}\,.\overrightarrow{dS} = \frac{q}{a^3}r^3$$

Or, $D \times 4\pi\, r^2 = \frac{q}{a^3}r^3 \qquad i.e., D = \frac{q}{4\pi}\frac{r}{a^3} \rightarrow E_{in} = \frac{q}{4\pi\epsilon_0\epsilon_r}\frac{r}{a^3}$

Hence the energy inside dielectric, $U_{inside} = \frac{1}{2}\iiint \vec{D}\,.\vec{E}_{in}\ d\tau$

$$= \frac{1}{2}\int_0^a \frac{q}{4\pi}\frac{r}{a^3} \times \frac{q}{4\pi\epsilon_0\epsilon_r}\frac{r}{a^3} \times 4\pi\, r^2\, dr = \frac{1}{2} \times \frac{q^2}{4\pi\epsilon_0\epsilon_r a^6} \times \int_0^a r^4\, dr$$

$$U_{inside} = \frac{1}{2} \times \frac{q^2}{4\pi\epsilon_0\epsilon_r a^6} \times \frac{a^5}{5} = \frac{q^2}{40\,\pi\,\epsilon_r\epsilon_0\,a} \qquad (1)$$

For $a \ge r < \infty$(point outside sphere)

$$\iint \vec{D}\,.\overrightarrow{dS} = Q_{free} \rightarrow \iint \vec{D}\,.\overrightarrow{dS} = q$$

$$\text{Or, } D \times 4\pi\, r^2 = q$$

$$i.e., D = \frac{q}{4\pi\, r^2} \,\&\, hence, E_{out} = \frac{D}{\epsilon_0} = \frac{q}{4\pi\epsilon_0 r^2}$$

Hence the energy outside is, $U_{outside} = \frac{1}{2}\epsilon_0 \iiint E_{out}{}^2\, d\tau$

$$U_{outside} = \frac{1}{2}\epsilon_0 \int_a^\infty \left(\frac{q}{4\pi\epsilon_0 r^2}\right)^2 4\pi\, r^2\, dr = \frac{1}{2}\epsilon_0 \times \left(\frac{q}{4\pi\epsilon_0}\right)^2 \times \int_a^\infty \frac{1}{r^2}\, dr$$

$$= \frac{1}{2}\epsilon_0 \times \left(\frac{q}{4\pi\epsilon_0}\right)^2 \times 4\pi \times \left[-\frac{1}{r}\right]_a^\infty = \frac{1}{2}\epsilon_0 \times \left(\frac{q}{4\pi\epsilon_0}\right)^2 \times \frac{4\pi}{a}$$

$$U_{outside} = \frac{q^2}{8\pi\,\epsilon_0\, a} \qquad (2)$$

From equations (1) & (2), we get

$$U_{inside}/U_{outside} = \frac{q^2}{40\pi\,\epsilon_r\epsilon_0\, a} \Big/ \frac{q^2}{8\pi\,\epsilon_0\, a} = \frac{1}{5\,\epsilon_r} = \frac{1}{10} = 0.1$$

Ratio = $\boldsymbol{U_{inside}/U_{outside} = 0.1}$ **Ans: (0.1)**

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